

# Introduction to Probability

independent events: where one outcome does not affect the other

dependent events: events that affect each other

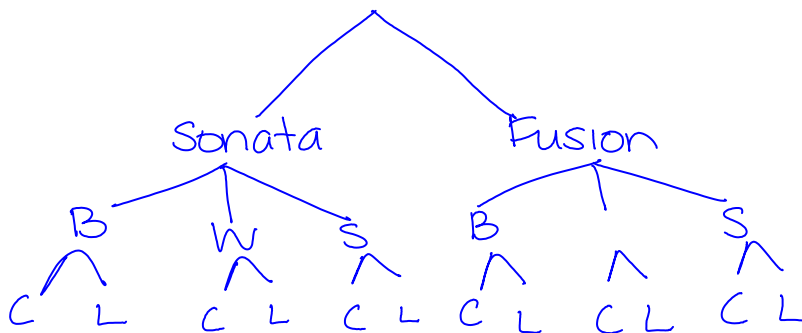
## Basic Counting Principle

Suppose one event can be chosen in  $p$  different ways, and another independent event can be chosen  $q$  different ways. Then the two events can be chosen successively  $p \cdot q$  ways.

Formula:  $p \cdot q \cdot r \dots$   
Illustration: tree diagram

Example 1: Kinkaid had to buy a new car. She had to choose between a Hyundai Sonata and a Ford Fusion. She then chose between black, white and silver. Finally, she chose between cloth and leather seats.

$$\frac{2}{\text{make}} \cdot \frac{3}{\text{color}} \cdot \frac{2}{\text{seats}} = 12 \text{ options}$$





Example 2: How many combinations of 7-digit phone numbers are there?

$$\underline{9} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 9,000,000$$

How many combinations of 7-digit numbers can you make if the first 3 must be 856?

$$\frac{1}{8} \cdot \frac{1}{5} \cdot \frac{1}{6} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 10,000$$

## Permutations

an arrangement of objects in a certain order

$$P(n, n) \text{ or } {}_n P_n = n!$$

$n$  objects taken  $n$  at a time  
choose to arrange all of the objects available

Example: how many ways can you arrange 5 picture frames on a shelf?

$$P(5, 5) \text{ or } {}_5 P_5 = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$P(n, r) \text{ or } {}_n P_r = \frac{n!}{(n-r)!}$$

$n$  objects chosen  $r$  at a time  
you choosing a certain number of objects ( $r$ )  
out of a larger group ( $n$ )

Example: how many different ways can

you seat students in a row of 4 if there are 30 students in the class?

$$P(30,4) \text{ or } {}_{30}P_4 = \frac{30!}{(30-4)!} = \frac{30!}{26!} = \frac{30 \cdot 29 \cdot 28 \cdot 27 \cdot \cancel{26 \cdot 25 \dots}}{\cancel{26 \cdot 25 \dots}}$$

$$= 657,720$$

Example 2: 6 people want to run the 4x100. How many different arrangements can be made to form the relay team?

$$P(6,4) \text{ or } {}_6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$$

$$= 360$$