

Exponential Functions

* to solve an equation with a variable as the exponent, take the log of both sides

* to solve an equation with e , use natural logs

Interest

annually \rightarrow 1 time per year
biannually \rightarrow 2 times per year
quarterly \rightarrow 4 times per year
monthly \rightarrow 12 times per year

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

A = amount

P = principal
(starting value)

r = rate

n = # of times per year

t = # of years

continuously compounded $A = Pe^{rt}$

Half-Life

$$y = a b^x$$

starting amount \rightarrow a
 $\frac{1}{2}$ \rightarrow b
 $\frac{x}{\text{length of half-life}}$ \rightarrow x

Growth

$$y = a b^x$$

growth factor \rightarrow b

Example 1: Interest

a) you invest \$10,000 in an account that pays 8%, compounded quarterly

How much will you have after 3 years?

$$A = 10000 \left(1 + \frac{.08}{4}\right)^{4 \cdot 3}$$

$$A = 10000(1.02)^{12} = \$12682.42$$

How long will it take to double?

$$\frac{20000}{10000} = \frac{10000}{10000} (1.02)^{4t} \rightarrow 2 = 1(1.02)^{4t}$$

$$2 = (1.02)^{4t}$$

$$\frac{\log 2}{\log 1.02} = 4t \frac{\log 1.02}{\log 1.02}$$

$$\frac{35.0028}{4} = \frac{4t}{4}$$

$$8.75 = t$$

years

b) You invest \$2000 in an account earning 6%, compounded monthly?

How much will be in the account after 2 years?

$$A = 2000 \left(1 + \frac{.06}{12}\right)^{12 \cdot 2} \rightarrow 2000(1.005)^{24}$$

$$\$2254.32$$

How long will it take to triple?

$$6000 = 2000(1.005)^{12t}$$

$$3 = (1.005)^{12t}$$

$$\frac{\log 3}{\log 1.005} = \frac{12t \log 1.005}{\log 1.005}$$

$$220.2713 = 12t$$

$$18.36 = t$$

years

c) A man invests \$6500 in an account

-) paying 6% interest, compounded continuously.

How much do you have after 3 years?

$$A = 6500 e^{(.06 \cdot 3)}$$
$$\$7781.91$$

How long will it be before you have \$9750?

$$9750 = 6500 e^{.06t}$$
$$1.5 = e^{.06t}$$
$$\ln 1.5 = .06t \quad \ln e = 1$$
$$.4055 = .06t$$
$$6.76 = t$$

years

Example 2: Exponential Growth/Decay

a) a sample of 1500 bacteria doubles every 30 minutes.

How much is present after 2 hours?

$$y = a b^x$$
$$1500(2)^{\frac{120}{30}} \rightarrow 1500(2)^4$$
$$y = 24000$$

b) Polonium-210 has a half-life of 140 days. You start with a 300 mg sample.

How much remains after 1 year

$$y = 300 \left(\frac{1}{2}\right)^{\frac{365}{140}}$$
$$y = 49.24 \text{ mg}$$

How long will it take for 200 mg to remain?

$$200 = 300 \left(\frac{1}{2}\right)^{\frac{t}{140}}$$
$$\frac{2}{3} = \left(\frac{1}{2}\right)^{\frac{t}{140}}$$

$$\frac{\log(2/3)}{\log(1/2)} = t/140 \quad \frac{\log(1/2)}{\log(1/2)}$$

$$.5850 = t/140$$

$$81.89 = t$$

or

$$81.90$$