

Law of Cosines

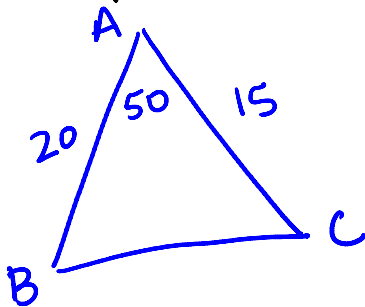
$$\begin{aligned}a^2 &= b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A \\b^2 &= a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos B \\c^2 &= a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos C\end{aligned}$$

Use Law of Cosines when given SSS or SAS.

If you start with Law of Cosines,
finish with it!! Don't switch to
Law of Sines.

Example 1: SAS

$$A = 50^\circ \quad b = 15 \quad c = 20$$



Find the third side first!

$$\begin{aligned}a^2 &= (20)^2 + (15)^2 - 2(20)(15)\cos(50) \\a^2 &= 239.327 \dots \\a &= \sqrt{\text{ANS}} \\a &= 15.47\end{aligned}$$

Now choose one of the missing angles.
We will find B first!

$$\begin{aligned}15^2 &= (20)^2 + (15.47)^2 - 2(20)(15.47)\cos B \\225 &= 400 + 239.3209 - 618.8 \cos B \\225 &= 639.3209 - 618.8 \cos B \\-639.3209 &= -639.3209 \\-414.3209 &= -618.8 \cos B \\-\frac{414.3209}{-618.8} &= \frac{-618.8 \cos B}{-618.8}\end{aligned}$$

$$.6695 \dots = \cos B$$

$$\cos^{-1}(\text{ANS}) = B$$

$$47.97 = B$$

Use Δ angle sum to find C

$$82.03 = C$$

Example 2: SSS $a=3$ $b=5$ $c=7$

Doesn't matter which angle you find first.

Angle A

$$3^2 = 5^2 + 7^2 - 2(5)(7)\cos A$$

$$9 = 25 + 49 - 70\cos A$$

$$9 = 74 - 70\cos A$$

$$\begin{array}{r} -74 \quad -74 \\ -65 = -70\cos A \end{array}$$

$$\begin{array}{r} -70 \quad -70 \end{array}$$

$$.928... = \cos A$$

$$\cos^{-1}(\text{ANS}) = A$$

$$21.79 = A$$

Angle B

$$5^2 = 3^2 + 7^2 - 2(3)(7)\cos B$$

$$25 = 9 + 49 - 42\cos B$$

$$25 = 58 - 42\cos B$$

$$\begin{array}{r} -58 \quad -58 \end{array}$$

$$-33 = -42\cos B$$

$$\begin{array}{r} -42 \quad -42 \end{array}$$

$$.7857 = \cos B$$

$$\cos^{-1}(\text{ANS}) = B$$

$$38.21 = B$$

Angle C Use Δ Angle Sum

$$21.79 + 38.21 + C = 180$$

$$C = 120$$